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THEORETICAL MODEL OF ACOUSTIC EMISSION UNDER MECHANICAL LOADING OF ROCKS IN THE MAXIMUM COMPACTION REGION

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A theoretical model of changes in acoustic emission activity in a geomaterial under continuous or stepwise mechanical loading is justified. Based on this model, the experimentally found laws of emission in the region of the maximum compaction of rock samples with different rates of mechanical loading of these samples are analyzed.

Key words: *acoustic emission activity, theoretical model, rock fracture, stepwise loading, linearly increasing loading.*

Introduction. Mechanical loading of rocks is accompanied by two interrelated simultaneous processes: formation of new defects of continuity and compaction caused by crack closure. One of these processes prevails at different stages of deformation. For geomonitoring practice, in particular, for evaluating long-time strength and predicting rock fracture, it is important to know the stage at which a transition occurs from prevailing compaction of the geomaterial to its softening. The region of such a transition is conventionally called the maximum compaction region. It can be identified on the basis of two acoustic-emission effects, which are most clearly manifested in plastic rocks and coal [1, 2]. The first effect implies that, as the sample loading is continuously increased, the minimum acoustic emission activity (AEA) is observed at the moment of the maximum compaction of the geomaterial. The second effect is manifested under stepwise loading of the sample as follows. At each stage of loading, the AEA value experiences a short-time increase and then decays exponentially following the dependence

$$E_i(t) = a_0 + a_1 \exp(-t/a_2), \qquad (1)$$

where a_0, a_1, and a_2 are the parameters characterizing the steady AEA, the AEA jump after loading at the next ith stage, and the time of AEA decay to the steady value. The state of the maximum compaction corresponds to the minimum value of the parameter a_2.

To correctly interpret the results of acoustic emission observations, including those under field conditions, it is necessary to develop a theoretical model that could suggest an explanation for the above-mentioned effects. In the present paper, we consider one possible variant of such a model based on the concept of the statistical nature of an individual act of fracture accompanied by acoustic emission. It is assumed that the (rock) material possesses a spectrum of parameters determining the fracture process.

1. Statistical Model. Acoustic emission effects arising in deformation of geomaterials are often explained with the use of models based on thermoactivation mechanisms of fracture [3–5]. According to the concept described in [4, 5], the fracture process is determined by consecutive breakdown of individual bonds between the corresponding structural elements. If $P = P(\sigma)$ is assumed to be the probability of an act of fracture of an individual bond under the action of the stress σ, and ω is the natural frequency of oscillations of structural elements connected by this

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bond, there will continuously appear ω situations where the bond is broken. The probability of fracture during the time dt is $P\omega\, dt$, and the probability that no breakdown will occurs during the time t is $(1-P)^{\omega t}$. If N_0 is the initial number of loaded bonds, the number of bonds retaining after the time t is $N_0(1-P)^{\omega t}$, and the number of bonds destroyed during the time interval from t to $t+dt$ is [4]

$$dN = N_0(1-P)^{\omega t} P\omega\, dt. \tag{2}$$

The ratio of the number of bonds destroyed by the time t to their initial number (dimensionless accumulated damage) is $1-(1-P)^{\omega t}$. Dependence (2) is valid for fracture processes at different scales with the characteristic time $1/\omega$.

We assume that the acoustic emission activity E, which is the number of acts of discrete acoustic emission per unit time, is proportional to the bond-breakdown rate. Then, following Eq. (2), we obtain the expression

$$E = \frac{dN}{dt} = N_0(1-P)^{\omega t} P\omega,$$

which is valid at constant stress.

Stepwise Increase in Stress. We consider the process of stepwise variation of loading with time. Let the sample experience the stress $\sigma(t_0)$ in the time interval from t_0 to t_1 and the stress $\sigma(t_1)$ from the time t_1 to the current time t. The probability of an individual act of fracture at the current time is

$$dN = N_0[1-P(\sigma(t_0))]^{\omega(t_1-t_0)}[1-P(\sigma(t_1))]^{\omega(t-t_1)} P(\sigma(t_1))\omega\, dt,$$

and

$$E(t) = N_0[1-P(\sigma(t_0))]^{\omega(t_1-t_0)}[1-P(\sigma(t_1))]^{\omega(t-t_1)} P(\sigma(t_1))\omega, \qquad t > t_1. \tag{3}$$

If the sample has experienced M jumps in stress since the beginning of the experiment, the duration of each jump being Δt, the rule for calculating the conditional probability of an individual act of fracture at the current time is

$$dN = N_0 \prod_{m=1}^{M} [1-P(\sigma(t_m))]^{\omega\Delta t} P(\sigma(t_M))\omega\, dt, \tag{4}$$

and

$$E(t) = N_0 \prod_{m=1}^{M} [1-P(\sigma(t_m))]^{\omega\Delta t} P(\sigma(t_M))\omega, \qquad t > t_M. \tag{5}$$

Continuous Loading. Let us find the limit of Eq. (5) as $\Delta t \to 0$, which corresponds to continuous loading. For this purpose, we divide the left and right sides of Eq. (5) by $N_0 P(\sigma(t_M))\omega$ and take its logarithm to obtain

$$\ln \frac{E(t)}{N_0 \omega P(\sigma(t_M))} = \omega\Delta t \sum_{m=1}^{M} \ln\,[1-P(\sigma(t_m))] N_0, \qquad t > t_M.$$

In passing to the limit $\Delta t \to 0$, we replace summation by integration:

$$\ln \frac{E(t)}{N_0 \omega P(\sigma(t))} = \omega \int_0^t \ln\,[1-P(\sigma(\tau))]\, d\tau.$$

In this case, the time evolution of AEA is determined by the formula

$$E(t) = N_0 \omega P(\sigma(t)) \exp\Big[\omega \int_0^t \ln\,[1-P(\sigma(\tau))]\, d\tau\Big]. \tag{6}$$

Normally, deriving such formulas is based on the property of smallness of the expression $1-[1-P(\sigma(t_m))]^{\omega\Delta t}$. With allowance for this property, the right side of Eq. (4) can be expanded with respect to the corresponding small parameter and can be written in the following form, only the principal terms being left:

$$E(t) = N_0\Big[1 - \sum_{m=0}^{M} P(\sigma(t_m))\omega\Delta t\Big] P(\sigma(t_M))\omega, \qquad t > t_M.$$

Using the transition to the limit $\Delta t \to 0$, we obtain the expression for continuous loading:

$$E(t) = N_0 \omega P(\sigma(t)) \Big[1 - \omega \int\limits_0^t P(\sigma(\tau))\, d\tau\Big]. \tag{7}$$

A comparison of the approximate expression (7) and a more accurate relation (6) shows that Eq. (7) is valid for low values of the dimensionless accumulated damage determined by the integrals in the right sides of Eqs. (6) and (7).

To derive the law of the AEA behavior for a particular material, it is necessary to prescribe the probability of the act of fracture as a function of stress $P(\sigma)$, which is typical for this particular material. The present model describes the behavior of materials in which all elements have identical strength characteristics. For real materials, such as rocks, which are inhomogeneous, we can naturally assume that there exists a certain distribution in strength of elements that compose the rock material. Dividing all elements into K groups in terms of strength and assuming that the behavior of each group is independent of the behavior of other groups, we can write Eq. (6) in the form

$$E(t) = \sum_{k=1}^{K} N_k \omega_k P_k(\sigma(t)) \exp\Big[\omega_k \int\limits_0^t \ln\left[1 - P_k(\sigma(\tau))\right] d\tau\Big]. \tag{8}$$

Performing the limiting transition corresponding to a smooth distribution of properties, we obtain the following expression from Eq. (8):

$$E(t) = \int\limits_0^{N_0} \omega(N) p(N, \sigma(t)) \exp\Big[\omega(N) \int\limits_0^t \ln\left[1 - p(N, \sigma(\tau))\right] d\tau\Big]\, dN. \tag{9}$$

As a parameter characterizing the strength of the group of elements considered, we use the fracture stress σ_0 typical of this group. Then, we have

$$E(t) = \int\limits_0^{\sigma_{\max}} Q(\sigma_0)\omega(\sigma_0) P(\sigma_0, \sigma(t)) \exp\Big[\omega(\sigma_0) \int\limits_0^t \ln\left[1 - P(\sigma_0, \sigma(\tau))\right] d\tau\Big]\, d\sigma_0. \tag{10}$$

Here the function $Q(\sigma) = dN/d\sigma$ characterizes the distribution density of elements in terms of their strength. The upper limit of integration in Eq. (10) can be set to infinity [in this case, the function $Q(\sigma)$ should vanish after a certain value].

Let the law $P(\sigma)$ be such that the element is not destroyed if the stress applied does not exceed a certain value. The upper limit of integration in Eq. (10) can be set equal to the acting stress σ. In addition, we can introduce the characteristic time of fracture $t_0 = 1/\omega$ instead of the frequency ω for convenience. Then, we obtain

$$E(t) = \int\limits_0^{\sigma} \frac{Q(\sigma_0)}{t_0(\sigma_0)} P(\sigma_0, \sigma(t)) \exp\Big[\frac{1}{t_0(\sigma_0)} \int\limits_0^t \ln\left[1 - P(\sigma_0, \sigma(\tau))\right] d\tau\Big]\, d\sigma_0. \tag{11}$$

Thus, in the general case, $E(t)$ is a functional depending on three parameters: distribution of strength $Q(\sigma_0)$, spectrum of characteristic times of fracture $t_0(\sigma_0)$, and spectrum of dependences of the probability of the act of fracture on the acting stress $P(\sigma_0, \sigma(\tau))$.

2. Dependence of the Fracture Probability on the Acting Stress in More Detail. We consider the case where the element is not destroyed if the stress applied does not exceed a certain value σ_0, i.e., $P(\sigma_0) = 0$ (with increasing stress σ, the probability of fracture increases and approaches unity). There exists an infinite set of functions of this form (e.g., the integral of the log-normal distribution, combination of powers, etc.). The choice of the function is determined by convenience and simplicity of expressions obtained. Hence, we choose a dependence in the form

$$P(\sigma_0, \sigma(t)) = \begin{cases} 1 - \exp\left[-a(\sigma_0)(\sigma(t) - \sigma_0)\right], & \sigma(t) > \sigma_0, \\ 0, & \sigma(t) < \sigma_0. \end{cases} \tag{12}$$

The function $a(\sigma)$ characterizes the growth rate of the fracture probability with increasing stress σ.

Substituting Eq. (12) into Eq. (11), we obtain the expression for an arbitrary dependence of loading on time. (The presence of a double integral in the resultant expression complicates its direct use.)

For the dependence chosen, we consider two cases: stepwise and constant-rate loading.

Stepwise Loading. We consider the case where the sample experiences the load σ_0 for a long time t_1 and then the load is instantaneously increased by $d\sigma$. To find the dependence $E(t-t_1)$, we use the intermediate expression (3). Similar to (9)–(11), we integrate this expression over the entire range of strength of its elements:

$$E(t) = \int\limits_0^{\sigma} \frac{Q(\sigma')}{t_0(\sigma')} [1 - P(\sigma_0)]^{t_1/t_0(\sigma_0)} [1 - P(\sigma')]^{(t-t_1)/t_0(\sigma')} P(\sigma')\, d\sigma' \qquad (t > t_1) \tag{13}$$

(σ' is the integration variable).

If the sample is retained under the stress σ_0 for a sufficiently long time $[t_1 \gg t_0(\sigma_0)]$, we can assume that almost all elements whose initial strength is lower than σ_0 will be destroyed. Then, Eq. (13) is simplified to

$$E(t) = \int\limits_{\sigma_0}^{\sigma} \frac{Q(\sigma')}{t_0(\sigma')} [1 - P(\sigma')]^{(t-t_1)/t_0(\sigma')} P(\sigma')\, d\sigma', \qquad t > t_1. \tag{14}$$

Substituting Eq. (12) into Eq. (14), we obtain the relation

$$E(t) = \int\limits_{\sigma_0}^{\sigma} \frac{Q(\sigma')}{t_0(\sigma')} \exp\Big(- a(\sigma_0)(\sigma' - \sigma_0)\, \frac{t - t_1}{t_0(\sigma')}\Big) \Big\{1 - \exp\left[-a(\sigma_0)(\sigma' - \sigma_0)\right]\Big\}\, d\sigma', \qquad t > t_1.$$

In the case of a fairly smooth variation of the functions $Q(\sigma)$, $t_0(\sigma)$, and $a(\sigma)$, they can be assumed to be constant within the integration interval. For convenience, in addition, we shift the time reference so that the stress increment occurs at the initial time $t = 0$:

$$E(t) = \frac{Q(\sigma)}{t_0(\sigma)} \int\limits_{\sigma_0}^{\sigma} \exp\Big(- a(\sigma_0)(\sigma' - \sigma_0)\, \frac{t}{t_0(\sigma')}\Big) \Big\{1 - \exp\left[-a(\sigma)(\sigma' - \sigma)\right]\Big\}\, d\sigma', \quad t > t_1. \tag{15}$$

Integration in Eq. (15) can be performed analytically, and the final result can be written as follows:

$$E(t) = \frac{Q}{a}\Big(\frac{\exp\left[-a(\sigma - \sigma_0)(1 + t/t_0)\right]}{t + t_0} - \frac{\exp\left[-a(\sigma - \sigma_0)t/t_0\right]}{t} + \frac{t_0}{t(t + t_0)}\Big), \quad t > t_1. \tag{16}$$

It follows from Eqs. (15) and (16) that the signal decay is not rigorously exponential and is characterized by a wide range of decay times. This could serve as a possible reason for a slower decay of the signal with time, which is observed in experiments. If the stress increment $\Delta\sigma = \sigma - \sigma_0$ is rather small, however, integral (15) in the first approximation can be calculated by the trapezium formula

$$E(t) \approx \Delta\sigma(E(\sigma) + E(\sigma_0))/2 = a_1(\sigma, \Delta\sigma) \exp\left[-t/a_2(\sigma, \Delta\sigma)\right], \qquad t > t_1,$$

where

$$a_1(\sigma, \Delta\sigma) = \Delta\sigma\, \frac{Q(\sigma)}{t_0(\sigma)}\, \frac{1 - \exp\left[-a(\sigma)\Delta\sigma\right]}{2} = \Delta\sigma\, \frac{Q(\sigma)P(\sigma)}{2t_0(\sigma)}, \qquad a_2(\sigma, \Delta\sigma) = \frac{t_0(\sigma)}{a(\sigma)\Delta\sigma}. \tag{17}$$

The value of $\Delta\sigma$ is normally constant in the course of experiments, whereas the model parameters (17) change.

Constant-Rate Loading. We consider a sample loaded with a constant rate

$$\sigma(t) = bt$$

(b is a constant coefficient). Substituting this expression into Eq. (12), and Eq. (12) into Eq. (11), we obtain an expression where the inner integral can be calculated analytically. As a result, we have

$$E(t) = \int\limits_0^{\sigma} \frac{Q(\sigma_0)}{t_0(\sigma_0)} \left[1 - \exp\left(-a(\sigma_0)(\sigma - \sigma_0)\right)\right] \exp\Big(- \frac{a(\sigma_0)}{2bt_0(\sigma_0)}\, (\sigma - \sigma_0)^2\Big)\, d\sigma_0.$$

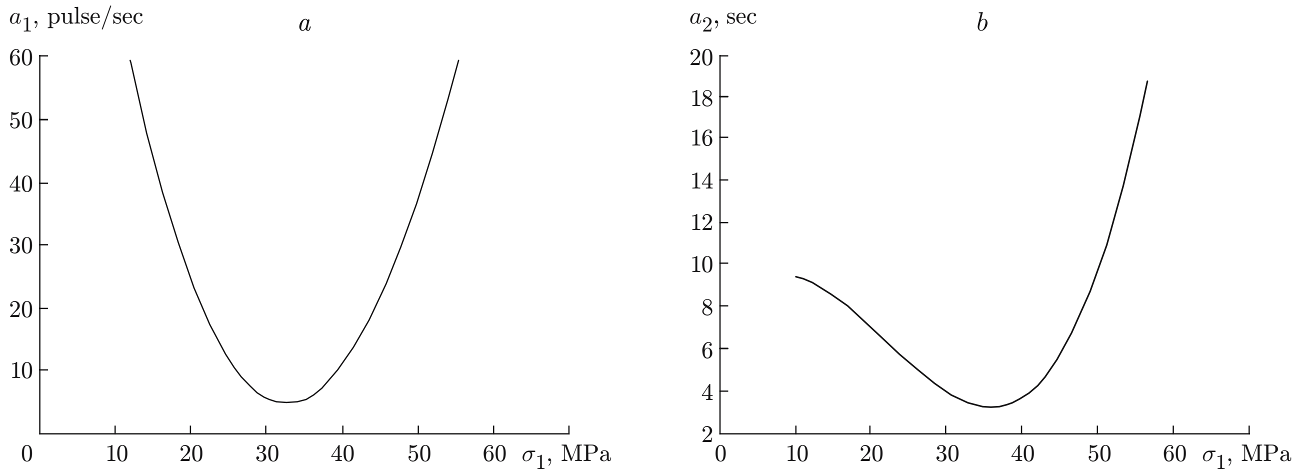

Fig. 1. AEA amplitude (a) and decay time (b) versus the axial load.

In some cases, e.g., if the loading is continuously increased with a constant rate, it is reasonable to write the AEA as a function of stress rather than a function of time:

$$E(\sigma) \equiv \frac{dN}{d\sigma} = \frac{dN}{dt}\,\frac{dt}{d\sigma} = \frac{E(t)}{b} = \frac{1}{b}\int_0^{\sigma} \frac{Q(\sigma_0)}{t_0(\sigma_0)}\,[1-\exp(-a(\sigma_0)(\sigma-\sigma_0))]\exp\Big(-\frac{a(\sigma_0)}{2bt_0(\sigma_0)}\,(\sigma-\sigma_0)^2\Big)\,d\sigma_0. \qquad (18)$$

Using the above-given notation and results, Eq. (18) can be written as

$$E(\sigma) = \frac{2}{b}\int_0^{\sigma} a_1(\sigma_0)\exp\Big(-\frac{(\sigma-\sigma_0)^2}{2ba_2(\sigma_0)}\Big)\,d\sigma_0.$$

3. Example of Calculation. We calculate the AEA with a continuously increasing load by using the function $a_2(\sigma)$ obtained on the basis of laboratory tests of coal samples under stepwise loading with a side pressure of 10 MPa.

Let two samples be loaded with a constant rate $\sigma(t) = bt$ ($b = 1$). For the first (reference) sample, the parameters of Eq. (1) are constant: $a_1 = 50$ and $a_2 = 10$; for the second sample (real coal sample), these parameters depend on stress as follows:

$$a_1 = 147 - 9.246\sigma + 0.166\sigma^2 + 0.000498\sigma^3; \qquad (19)$$

$$a_2 = 7.626 + 0.496\sigma - 0.0378\sigma^2 + 0.000573\sigma^3. \qquad (20)$$

Dependences (19) and (20) obtained by approximating appropriate experimental data by third-power polynomials are plotted in Fig. 1a and Fig. 1b, respectively.

Figure 2 shows the AEA as a function of stress for a real coal sample and for the reference sample with constant properties. It follows from Fig. 2 that the law of AEA variation for a continuous increase in loading, which is consistent with experimental data, can be calculated by using the model proposed here with allowance for AEA variation under stepwise loading.

In addition to two acoustic emission effects described above, which are manifested with a continuous and stepwise increase in loading, the model developed offers an explanation for another effect observed in experiments. This effect is manifested in specific features of AEA variation with different growth rates of loading (Fig. 3): as the loading growth rate increases, the ratio of the maximum to the minimum AEA value in the maximum compaction region decreases. The degree of manifestation of this effect is determined by the ratio between the time during which the increasing load passes the stage of the maximum compaction and the time of AEA decay after application of loading at the previous loading stage. For moderate loading rates, with the time of passing the maximum compaction stage being sufficiently large as compared with the time of the residual effect of processes of the previous stage, there is a clearly expressed AEA minimum. If the loading rate is higher than that in the previous case, the time needed

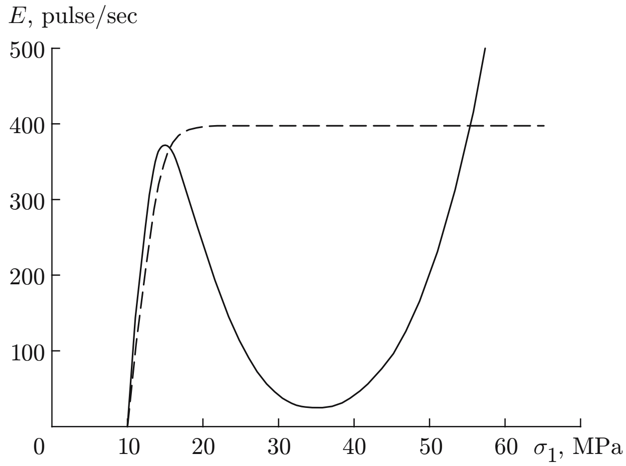

Fig. 2

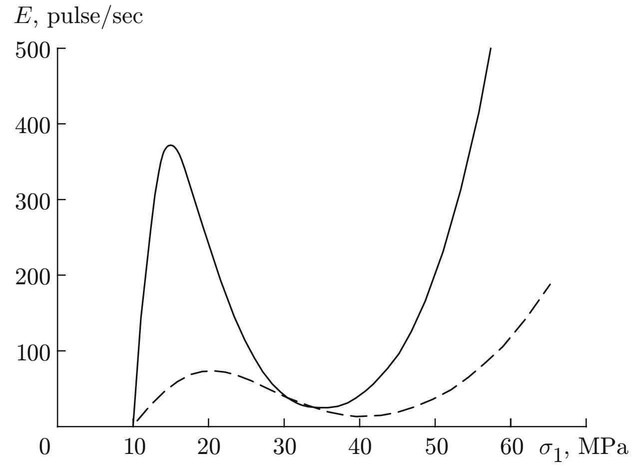

Fig. 3

Fig. 2. AEA versus the axial stress in the case of a linear increase in loading: the solid and dashed curves refer to the real coal sample and to the reference sample with constant properties, respectively.

Fig. 3. AEA versus the axial stress with the growth rate of loading $b = 1$ (solid curve) and $b = 10$ (dashed curve).

to pass the maximum compaction stage becomes commensurable with the time of the residual effect of the previous stage or shorter than the latter. As a result, the masking action of the AEA preceding the maximum compaction at the loading stage starts manifesting. In the limiting case, this "masking" can make the AEA minimum in the maximum compaction state disappear.

Conclusions. The theoretical model developed on the basis of postulates of the statistical theory of strength allows one to explain the anomaly of acoustic emission observed under continuous and stepwise mechanical loading of rocks in the maximum compaction region and also the influence of the load-variation rate on these anomalous features.

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